

Event by Event fluctuations and Inclusive Distributions

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Abstract

Event-by-event observables are compared with conventional inclusive measurements. We find that moments of event-by-event fluctuations are closely related to inclusive correlation functions. Implications for upcoming heavy ion experiments are discussed.

1. It is now widely recognized that studies of event-by-event fluctuations observed in high energy multi-particle reactions may become an important tool in attempts to understand the underlying dynamics of ultrarelativistic heavy ion collisions [1] - [7]. It has for instance been proposed that the measurement of event-by-event fluctuations of the temperature via e.g. the transverse momentum spectrum could provide information about the heat capacity of the system generated

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in these collisions [6, 7]. Also, by investigating event-by-event fluctuations one may be able to determine the degree of thermalization of the system [1, 2] or to select distinct event classes. The analysis of heavy ion collisions on an event-by-event basis has been pioneered by the NA49 experiment. First, preliminary results [5] seem to indicate that the observed fluctuation in the mean transverse momentum as well as the kaon to pion ratio are of Gaussian shape.

It seems, therefore, interesting to study in some detail what is the information content of such measurements and to what extent they actually differ from the more conventional treatment of particle spectra. In the present paper we discuss the relation of event-by-event fluctuations to the standard inclusive (multi-particle) distributions. Our conclusions can be summarized as follows: Moments of event-by-event fluctuations of any (multi-particle) observable can be expressed in terms of inclusive distributions, provided the inclusive distributions are known up to twice the order of the observable under consideration. For instance, in order to express the dispersion of the event-by-event distribution of a single particle observable, one needs to know the two-particle inclusive distribution etc. Fluctuations of ratios of observables, on the other hand, cannot simply be expressed in terms of ratios of inclusive measurements. In particular, to obtain moments of an observable corresponding to an "intensive" quantity (in the thermodynamic limit) it is necessary to know the inclusive distributions in narrow multiplicity intervals. However, as long as the central limit theorem can be applied, i.e. the observables involved are dominated by independent single particle emission, and the observed multiplicities are reasonably high, the knowledge of inclusive distribution of twice the order of the observables under consideration again is sufficient.

2. Let us consider a variable $x(p)$ which depends on momentum of one particle. We shall discuss event-by-event fluctuations of the quantity

$$S(x) = \sum_{i=1}^N x(p_i) \equiv \sum_{i=1}^N x(i) \quad (1)$$

where N is the multiplicity of the event.

The *event averaged* moments of this quantity can be expressed as

$$\langle S^k \rangle = \frac{1}{M} \sum_{m=1}^M \sum_{i_1=1}^{N_m} \dots \sum_{i_k=1}^{N_m} x_m(i_1) \dots x_m(i_k) \quad (2)$$

where m labels the different events and M is their total number. N_m is the multiplicity of the event labelled by m .

On the other hand, the moments of $x(p)$ calculated from n -particle inclusive distribution $\rho_n(p_1, \dots, p_n)$ are defined as

$$\int dp_1 \dots dp_n \rho_n(p_1, \dots, p_n) [x(p_1)]^{k_1} \dots [x(p_n)]^{k_n} = \frac{1}{M} \sum_{m=1}^M \sum_{i_1=1}^{N_m} \dots \sum_{i_n=1}^{N_m} [x_m(i_1)]^{k_1} \dots [x_m(i_n)]^{k_n} \quad (3)$$

where the sums over $i_1 \dots i_n$ include only the terms for which all indices $i_1 \dots i_n$ are different from each other.

One sees immediately that (2) and (3) are related.

$$\langle S \rangle = \int dp \rho_1(p) x(p) \quad (4)$$

$$\langle S^2 \rangle = \int dp_1 dp_2 \rho_2(p_1, p_2) x(p_1) x(p_2) + \int dp \rho_1(p) [x(p)]^2 \quad (5)$$

$$\begin{aligned} \langle S^3 \rangle &= \int dp_1 dp_2 dp_3 \rho_3(p_1, p_2, p_3) x(p_1) x(p_2) x(p_3) \\ &\quad + 3 \int dp_1 dp_2 \rho_2(p_1, p_2) x(p_1) [x(p_2)]^2 \\ &\quad + \int dp \rho_1(p) [x(p)]^3 \end{aligned} \quad (6)$$

Similar formulae can be derived for higher moments of S .

We have thus established the relation between inclusive measurements and event-by-event fluctuations for single particle observables [1], such as e.g. the transverse momentum, particle abundances [2] etc.

3. The same argument can be constructed for variables which depend on two or more particle momenta. In particular the fluctuations of Hanbury-Brown Twiss (HBT) two particle correlations belong to this class. They are of practical interest and will be investigated in future heavy ion experiments [8]. Here we will restrict the argument to two particles but it can be readily extended to multiparticle correlations. Consider a variable $y = y(p, p')$. We calculate the event by

event fluctuation of the quantity

$$T \equiv \sum_{i=1}^N \sum_{j=1}^N y(p_i, p_j) \quad (7)$$

where N is the multiplicity of the event and the sum runs only for $i \neq j$ (y is not defined for $i = j$).

Similarly as before we thus can write

$$\langle T^k \rangle = \frac{1}{M} \sum_{m=1}^M \sum_{i_1=1}^{N_m} \sum_{j_1=1}^{N_m} \dots \sum_{i_k=1}^{N_m} \sum_{j_k=1}^{N_m} y_m(i_1, j_1) \dots y_m(i_k, j_k) \quad (8)$$

Consequently we obtain

$$\langle T \rangle = \int dp_1 dp_2 \rho_2(p_1, p_2) y(p_1, p_2) \quad (9)$$

and

$$\begin{aligned} \langle T^2 \rangle &= \int dp_1 dp_2 dp_3 dp_4 \rho_4(p_1, p_2, p_3, p_4) y(p_1, p_2) y(p_3, p_4) \\ &\quad + 4 \int dp_1 dp_2 dp_3 \rho_3(p_1, p_2, p_3) y(p_1, p_2) y(p_1, p_3) \\ &\quad + 2 \int dp_1 dp_2 \rho_2(p_1, p_2) [y(p_1, p_2)]^2 \end{aligned} \quad (10)$$

and similarly for higher moments. Thus for any (multiparticle) observable event-by-event fluctuations can be re-expressed in terms of inclusive multiparticle distribution. The multiparticle distributions need to be known up to twice the order of the observable under consideration.

4. The argument of Sections 2 and 3 shows that the inclusive measurements give a precise information on event-by-event fluctuations of the quantities S and T defined by (1) and (7). As shown in [1, 2, 3] they may be very useful in investigation of the properties of the multiparticle system. It is also often interesting, however, to discuss the averages, as they resemble intensive variables in the thermodynamic limit:

$$s \equiv S/N; \quad t \equiv \frac{T}{N(N-1)} \quad (11)$$

Unfortunately, they can only be obtained if one knows the inclusive distribution separately for each multiplicity or in other words if one knows the inclusive distributions of the entire ratios and their second moments. The relevant formulae are easily obtained from those in section 2 and 3.

An approximate estimate can, however, be obtained from sole inclusive spectra, provided the correlations between the produced particles are not too strong. This can be seen as follows.

If the correlations are not overwhelmingly strong, the dispersion of S at fixed N is likely to follow the central limit theorem, i.e.

$$D_N^2 \equiv \langle S^2 \rangle_N - \langle S \rangle_N^2 \simeq N\delta^2 \quad (12)$$

where δ is a constant, independent of N . Assume furthermore that the average value of x does not depend on multiplicity. Then we have

$$\langle S \rangle_N \equiv N \langle s \rangle_N = N\sigma \quad (13)$$

where σ does not depend on multiplicity. Consequently,

$$\langle S^2 \rangle_N = N^2\sigma^2 + N\delta^2 \quad (14)$$

(13) and (14) can be now rewritten in terms of inclusive quantities:

$$\langle S \rangle \equiv \sum P_N \langle S \rangle_N \simeq \langle N \rangle \sigma \quad (15)$$

$$\langle S^2 \rangle \equiv \sum P_N \langle S^2 \rangle_N = \langle N^2 \rangle \sigma^2 + \langle N \rangle \delta^2 \quad (16)$$

Using 4 and 5 we can thus find σ and δ^2 from the inclusive measurements, provided we know $\langle N^2 \rangle$ and $\langle N \rangle$. But they are actually known, because they can be obtained from inclusive spectra

$$\langle N \rangle = \int \rho(p) dp; \quad \langle N(N-1) \rangle = \int dp_1 dp_2 \rho(p_1, p_2) \quad (17)$$

The question now is can we express $\langle s \rangle$ and $\langle s^2 \rangle$ in terms of σ and δ^2 . The answer is: almost.

We have:

$$\langle s \rangle_N = \sigma \quad \rightarrow \langle s \rangle \equiv \sum P_N \langle s \rangle_N = \sigma \quad (18)$$

This is easy. But from (14) we have

$$\langle s^2 \rangle_N \equiv \langle S^2 \rangle_N / N^2 = \sigma^2 + \delta^2 / N \quad (19)$$

and thus

$$\langle s^2 \rangle - \langle s \rangle^2 = \delta^2 \langle \frac{1}{N} \rangle \quad (20)$$

and one sees that we need the average of $1/N$ which is not possible to obtain if only two-arm spectrometer is available. But this is not a serious problem in practice.

Indeed, as long as the fluctuations in N are small $\langle \frac{1}{N} \rangle$ to a good approximation is given by

$$\langle \frac{1}{N} \rangle = \frac{1}{M} \sum_{i=1}^M \frac{1}{N_i} \simeq \frac{1}{\langle N \rangle} \left(1 + \frac{\langle (N - \langle N \rangle)^2 \rangle}{\langle N \rangle^2} \right) \quad (21)$$

For typical ‘central’ trigger conditions at a 200 AGeV $Pb + Pb$ collisions, there are about 200 negatively charged particles per unit rapidity at mid rapidity. Assuming that a reasonable two arm spectrometer covers about half a unit of rapidity the fluctuations in the number of particles are, assuming simple statistics, about

$$\frac{\langle (N - \langle N \rangle)^2 \rangle}{\langle N \rangle^2} \simeq \frac{1}{\langle N \rangle} \simeq 1\% \quad (22)$$

5. We would like to close this paper with the following comments.

(i) Our argument shows that there is a close link between the event-by-event analysis and the inclusive measurements. It follows that, contrary to common belief, accuracy of the event-by-event analysis hardly depends on event multiplicity and thus can be useful even for low multiplicity events. Of course the approximation for $\langle 1/N \rangle$ presented above gets less reliable with small multiplicities. Also the applicability of the central limit theorem is less reliable for small multiplicities. However, for fixed multiplicity, our arguments are rigorous and thus the differences between event-by-event analysis and the inclusive measurement can be minimized by narrow multiplicity cuts. Furthermore, one should remember that these differences are solely due to the fact that one wants to express the ratio of observables in terms of ratios of inclusive expectation values. In principle this is not necessary and the ratio itself can be obtained via an inclusive measurement. In that case our arguments again are rigorous.

(ii) It is important to realize that the multiplicity N which enters the formulae of this paper is *not necessarily* the total multiplicity of the event. It is the multiplicity of the particles *of interest* and refers

to the specific phase-space region under investigation. In that case, of course, also the inclusive densities refer to the same particles. This means in particular that it is not necessary to use a 4π detector in order to apply the argument of the present paper.

(iii) In practice, the formulae of sections 2 and 3 can only be useful for the low rank moments. Nevertheless such information is very often of great interest [1, 2, 3]. Needless to say, they cannot be used to investigate the tail of the event-by-event distribution, i.e. for search of rare, exotic events.

(iv) In principle any large acceptance spectrometer, such as NA49, can also be used as two arm spectrometer. Thus, the above fluctuations can be extracted on an event-by-event basis as well as in the inclusive way and it would be interesting to compare the results of both approaches.

In conclusions, we have written down the explicit relation between the event-by-event analysis and inclusive multi-particle measurements. The relation is rather straightforward and can be useful when applied in experiments where no direct event-by-event measure is possible.

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